Web Appendix to "Estimating Labor-Supply Elasticities with Joint Borrowing Constraints of Couples"

by C. Bredemeier, J. Gravert, and F. Juessen

A Generalizing the relation between the borrowing-constraint bias and the relative earnings contribution

In this Appendix, we show that the estimation bias due to borrowing constraints in Altonji (1986) regressions is decreasing in an individual's contribution to household earnings for an arbitrary data frequency. For the general case, the estimation bias cannot be derived in closed form but we can express it in a comprehensive way as a function of endogenous moments as also done by Domeij and Flodén (2006). First, we show that a standard Altonji (1986) regression is subject to a downward bias also in our two-earner model. This result and its derivation are very similar to the corresponding result in the bachelor model of Domeij and Flodén (2006). Then, we show that this bias is the less important the less an individual contributes to household earnings. This second result critically requires the double-earner structure.

We start by deriving the true relation between hours growth and expected wage growth in our model. Taking logs and first differences of the labor-supply conditions (8) and (9), we obtain (for $i = 1, 2$)

$$\Delta \ln w_i' = \frac{1}{\eta} \cdot \Delta \ln n_i - \Delta \ln \lambda'. \tag{44}$$

A log-linear approximation of the Euler equation (7) implies

$$\Delta \ln \lambda' = -\ln \beta - \ln (1 + r) - \frac{\phi}{\lambda} + \xi', \tag{45}$$

where $\xi' = \ln \lambda' - \mathrm{E} \ln \lambda'$ denotes an expectation error. Inserting (45) into (44) and rearranging yields

$$\Delta \ln n_i' = \eta \cdot \Delta \ln w_i' - \eta \cdot \ln \beta - \eta \cdot \ln (1 + r) - \eta \cdot \frac{\phi}{\lambda} + \eta \cdot \xi'. \tag{46}$$

Note that this expression nests both relations, for unconstrained and constrained households, respectively, derived in the main text. For households unaffected by borrowing constraints, $\phi = 0$, such that (46) simplifies to (13) for $i = 1, 2$, when using $\Delta \ln w_i' = \Delta \mathrm{E} \ln w_i' + \omega_i'$. For borrowing-constrained households, the multiplier ratio ϕ/λ can be substituted to obtain (17). Further, (46) also covers households who transition between being borrowing constrained and unconstrained.

As pointed out by Altonji (1986), the residual ξ' is correlated with wage growth $\Delta \ln w_i'$ but not with expected wage growth $\mathrm{E} \Delta \ln w_i'$. As in Section 3.1, one therefore applies a decomposition of $\Delta \ln w_i'$ into $\mathrm{E}\,\Delta \ln w_i'$ and an unexpected component ω_i'. However, the multiplier ratio ϕ/λ which measures the household's willingness to borrow affects labor supply but is not observable in empirical data and is therefore part of the combined residual. This causes the estimation bias due to borrowing constraints because the willingness to borrow is correlated with wage growth.[40]

We will now show that the associated estimation bias differs depending on an individual's earner role in the household. In our derivations, we distinguish between two cases regarding the observability of expected wage growth. First, when using synthetic data, one can calculate the expected wage change $\mathrm{E}\,\Delta \ln w_i'$ from the particular wage process assumed. In the quantitative evaluations of our model, we assume an AR(1) process with fixed effects, $w_i' = \psi_i + z_i$, $z_i' = \rho \cdot z_i + \varepsilon_i'$, where ψ_i is a fixed effect, ρ determines persistence and ε is a wage-rate shock. We also apply this assumption here. Thus, we can calculate expected wage growth as

$$\mathrm{E}\,\Delta \ln w_i' = \mathrm{E}(z_i' - z_i) = \mathrm{E}((\rho - 1) \cdot z_i + \varepsilon_i') = (\rho - 1) \cdot z_i. \tag{47}$$

Second, in real-world data, expected wage changes can be obtained through a first-stage regression using variables as regressors which are known to the agent in advance, see MaCurdy (1981), Altonji (1986), and Keane (2011). In the following, we cover both cases and derive important properties of the results of Altonji (1986) regressions in our model.

Proposition 1 *When expected wage growth $\mathrm{E}\,\Delta \ln w'$ is known, the percentage bias in the estimate for the Frisch elasticity in an Altonji (1986) regression is*

$$bias = \frac{\widehat{\eta} - \eta}{\eta} = -\frac{1+\rho}{(1-\rho) \cdot \sigma_\varepsilon^2} \cdot \operatorname{cov}\left(\phi/\lambda, \mathrm{E}\,\Delta \ln w'\right).$$

If expected wage growth is identified using an instrument y, the bias is

$$\frac{\widehat{\eta} - \eta}{\eta} = -\frac{1}{\widehat{\gamma}^2 \cdot \operatorname{var}(y)} \cdot \left(\operatorname{cov}\left(\phi/\lambda, \Delta \ln w'\right) - \operatorname{cov}\left(\phi/\lambda, \nu\right)\right),$$

where $\widehat{\gamma}$ is the estimated coefficient on the instrument in the first-stage regression and ν is the residual from the first-stage regression.

Proof. If expected wage changes are known to the econometrician, the labor-supply regression is

$$\Delta \ln n' = const + \eta \cdot \mathrm{E}\,\Delta \ln w' + u$$

[40] This problem is not resolved by using expected wage-rate changes as regressors. The willingness to borrow is correlated with precisely the expected component of wage changes as expected wage growth induces households to wish to front-load consumption through borrowing.

and the estimate $\widehat{\eta}$ is

$$\widehat{\eta}=\frac{\operatorname{cov}\left(\Delta\ln n',\mathrm{E}\,\Delta\ln w'\right)}{\operatorname{var}\left(\mathrm{E}\,\Delta\ln w'\right)}=\frac{\operatorname{cov}\left(\Delta\ln n',(\rho-1)\,z\right)}{\operatorname{var}\left((\rho-1)\,z\right)}=\frac{\operatorname{cov}\left(\Delta\ln n',z\right)}{(\rho-1)\cdot\operatorname{var}(z)}. \tag{48}$$

We rearrange the true labor-supply relation (46) to

$$\begin{aligned}\Delta\ln n'&=\kappa+\eta\cdot\Delta\ln w'-\eta\cdot\frac{\phi}{\lambda}+\widetilde{\xi}'=\kappa+\eta\cdot\left(z'-z\right)-\eta\cdot\frac{\phi}{\lambda}+\widetilde{\xi}'\\&=\kappa+\eta\cdot(\rho z+\varepsilon-z)-\eta\cdot\frac{\phi}{\lambda}+\widetilde{\xi}'=\kappa+\eta\cdot((\rho-1)\,z+\varepsilon)-\eta\cdot\frac{\phi}{\lambda}+\widetilde{\xi}',\end{aligned}$$

where $\kappa=-\eta\cdot\ln\beta-\eta\cdot\ln(1+r)$ and $\widetilde{\xi}'=\eta\cdot\xi'$. Hence, the covariance between $\Delta\ln n'$ and z is

$$\begin{aligned}\operatorname{cov}\left(\Delta\ln n',z\right)&=\operatorname{cov}\left(\eta\cdot(\rho-1)\,z-\eta\cdot\frac{\phi}{\lambda},z\right)\\&=\eta\cdot(\rho-1)\cdot\operatorname{var}(z)-\eta\cdot\operatorname{cov}\left(\frac{\phi}{\lambda},z\right).\end{aligned}$$

Inserting this into (48) gives the estimate as

$$\widehat{\eta}=\frac{\eta\cdot(\rho-1)\cdot\operatorname{var}(z)-\eta\cdot\operatorname{cov}\left(\frac{\phi}{\lambda},z\right)}{(\rho-1)\cdot\operatorname{var}(z)}=\eta-\frac{\eta\cdot\operatorname{cov}\left(\frac{\phi}{\lambda},z\right)}{(\rho-1)\operatorname{var}(z)}.$$

The percentage bias then is

$$\begin{aligned}\frac{\widehat{\eta}-\eta}{\eta}&=-\frac{\operatorname{cov}\left(\frac{\phi}{\lambda},z\right)}{(\rho-1)\operatorname{var}(z)}=-\frac{\operatorname{cov}\left(\phi/\lambda,\mathrm{E}\,\Delta\ln w'/(\rho-1)\right)}{(\rho-1)\operatorname{var}(z)}\\&=-\frac{\operatorname{cov}\left(\phi/\lambda,\mathrm{E}\,\Delta\ln w'\right)}{(\rho-1)^2\cdot\operatorname{var}(z)}=-\frac{1+\rho}{(1-\rho)\cdot\sigma_\varepsilon^2}\cdot\operatorname{cov}\left(\phi/\lambda,\mathrm{E}\,\Delta\ln w'\right),\end{aligned} \tag{49}$$

where the last step uses $\operatorname{var}(z)=1/\left(1-\rho^2\right)\cdot\sigma_\varepsilon^2$ and $1-\rho^2=(1+\rho)\cdot(1-\rho)$.

If expected wage-rate changes are identified based on a first-stage regression, $\Delta\ln w'=const_1+\gamma\cdot y+\nu$, where y is an instrument, the first-stage results are

$$\widehat{\gamma}=\frac{\operatorname{cov}\left(\Delta\ln w',y\right)}{\operatorname{var}(y)},\ \ \widehat{\Delta\ln w'}=const_1+\frac{\operatorname{cov}\left(\Delta\ln w',y\right)}{\operatorname{var}(y)}\cdot y.$$

Then, the second-stage regression is

$$\Delta\ln n'=const+\eta\cdot\frac{\operatorname{cov}\left(\Delta\ln w',y\right)}{\operatorname{var}(y)}\cdot y+u,$$

where $const$ includes $\eta\cdot const_1$ and the estimated coefficient is

$$\widehat{\eta}=\frac{\operatorname{cov}\left(\frac{\operatorname{cov}(\Delta\ln w',y)}{\operatorname{var}(y)}\cdot y,\Delta\ln n'\right)}{\operatorname{var}\left(\frac{\operatorname{cov}(\Delta\ln w',y)}{\operatorname{var}(y)}\cdot y\right)}=\frac{\frac{\operatorname{cov}(\Delta\ln w',y)}{\operatorname{var}(y)}\cdot\operatorname{cov}\left(y,\Delta\ln n'\right)}{\left(\frac{\operatorname{cov}(\Delta\ln w',y)}{\operatorname{var}(y)}\right)^2\cdot\operatorname{var}(y)}=\frac{\operatorname{cov}\left(y,\Delta\ln n'\right)}{\operatorname{cov}\left(\Delta\ln w',y\right)}. \tag{50}$$

We rearrange the true relation (46) to

$$\Delta \ln n' = \kappa + \eta \cdot \Delta \ln w' - \eta \cdot \frac{\phi}{\lambda} + \widetilde{\xi}' = \kappa + \eta \cdot (const_1 + \gamma \cdot y + \nu) - \eta \cdot \frac{\phi}{\lambda} + \widetilde{\xi}',$$

which uses the notation from above. Hence, the covariance between the dependent variable and the instrument is

$$\mathrm{cov}\left(\Delta \ln n', y\right) = \eta\gamma \cdot \mathrm{var}\,(y) - \eta \cdot \mathrm{cov}\,(y, \phi/\lambda)\,.$$

Expressing the instrument as $y = (\Delta \ln w' - const_1 - \nu)/\widehat{\gamma}$, we can state that

$$\mathrm{cov}\,(y, \phi/\lambda) = \frac{1}{\widehat{\gamma}} \cdot \mathrm{cov}\left(\Delta \ln w', \phi/\lambda\right) - \frac{1}{\widehat{\gamma}} \cdot \mathrm{cov}\,(\nu, \phi/\lambda)\,.$$

We can use the first-stage results to determine the covariance in the denominator of the estimate $\widehat{\eta}$ in (50) as

$$\mathrm{cov}\left(\Delta \ln w', y\right) = \widehat{\gamma} \cdot \mathrm{var}\,(y)\,.$$

Inserting this into (50) gives

$$\widehat{\eta} = \frac{\eta\gamma\,\mathrm{var}\,(y) - \eta \cdot \mathrm{cov}\,(y, \phi/\lambda)}{\widehat{\gamma} \cdot \mathrm{var}\,(y)} = \eta - \eta \cdot \frac{\mathrm{cov}\,(\Delta \ln w', \phi/\lambda) - \mathrm{cov}\,(\nu, \phi/\lambda)}{\widehat{\gamma}^2 \cdot \mathrm{var}\,(y)}.$$

Hence, the percentage bias is

$$\frac{\widehat{\eta} - \eta}{\eta} = -\frac{1}{\widehat{\gamma}^2 \cdot \mathrm{var}\,(y)} \cdot \left(\mathrm{cov}\left(\Delta \ln w', \phi/\lambda\right) - \mathrm{cov}\,(\nu, \phi/\lambda)\right)$$

as stated in the proposition. ■

For the following analytical results, we consider a simplified version of the model, where we assume $\eta_1 = \eta_2 = \eta$, $\alpha_1 = \alpha_2 = 2$, $\mu = 1/2$, $\rho = 0$, and $\mathrm{var}\,(\varepsilon_1) = \mathrm{var}\,(\varepsilon_2)$. Importantly, the assumptions on the preference weights α and μ imply that differences in long-run earner roles stem solely from differences in the wage fixed effects ψ. The higher a spouse's fixed effect, the more this person contributes to household earnings (see Lemma 3 below). Therefore, it is convenient to first perform comparative statistics in the fixed effects ψ and to transfer the results to relative earner roles thereafter.

As an intermediate step, we can state that the multiplier ratio in the true labor-supply relation (46) is weakly decreasing in a symmetric, increasing function of spouses' wage levels:

Lemma 1 *Define $\Lambda = w_1^{1+\eta} + w_2^{1+\eta}$. The multiplier ratio ϕ/λ is weakly decreasing in Λ,*

$$\frac{\partial\,(\phi/\lambda)}{\partial \Lambda} \leq 0.$$

Proof. Consider a period with state variables a, $w_1 = Z_1$, and $w_2 = \Psi_2 \cdot Z_2$. Using the first-order condition for consumption (7), the multiplier ratio ϕ/λ is

$$\frac{\phi}{\lambda} = \frac{\lambda - \beta\,(1+r)\,\mathrm{E}\,\lambda'}{\lambda} = 1 - \frac{\beta \cdot (1+r) \cdot \mathrm{E}\,\lambda'}{\lambda}.$$

Obviously, this ratio is zero if the household is not borrowing constrained in the current period, i.e., if $a' > 0 \Leftrightarrow \phi = 0$. Since z_2 and z_1 are i.i.d., a borrowing-constrained household expects to enter the next period with state variables $a' = z_1' = z_2' = 0$. Hence, we can consider $\mathrm{E}\,\lambda'$ in case of being borrowing constrained as a constant, which we denote by $\overline{\lambda}$.

Taken together, we can express the multiplier ratio as

$$\frac{\phi}{\lambda} = \max\left[1 - \beta \cdot (1+r) \cdot \frac{\overline{\lambda}}{\widetilde{\lambda}}, 0\right].$$

It is weakly increasing in the Lagrange multiplier on the current period's budget constraint $\widetilde{\lambda}$, $\partial\,(\phi/\lambda)\,/\partial\widetilde{\lambda} \leq 0$.

When the household is borrowing constrained, the Lagrange multiplier on the borrowing constraint can be determined from the remaining first-order conditions for the current period:

$$\begin{aligned}
n_2^{1/\eta} &= \widetilde{\lambda} \cdot w_2 = \widetilde{\lambda} \cdot Z_2 \cdot \Psi_2,\\
n_1^{1/\eta} &= \widetilde{\lambda} \cdot w_1 = \widetilde{\lambda} \cdot Z_1,\\
\widetilde{\lambda} &= c^{-\sigma},\\
c &= w_2 \cdot n_2 + w_1 \cdot n_1 + a\\
&= Z_2 \cdot \Psi_2 \cdot n_2 + Z_1 \cdot n_1 + a,
\end{aligned}$$

where the final condition uses $a' = 0$. Combining all conditions yields

$$\left(\widetilde{\lambda}\right)^{-1/\sigma} - \Lambda \cdot \left(\widetilde{\lambda}\right)^{\eta} - a = 0,$$

where $\Lambda = W_1^{1+\eta} + W_2^{1+\eta} = Z_2^{1+\eta} \cdot \Psi_2^{1+\eta} + Z_1^{1+\eta}$. Defining the left-hand side of this expression as F and applying the implicit-function theorem gives

$$\frac{\partial\widetilde{\lambda}}{\partial\Lambda} = -\frac{\partial F/\partial\Lambda}{\partial F/\partial\widetilde{\lambda}} = -\frac{-\widetilde{\lambda}^{\eta}}{-\left(\frac{1}{\sigma}\left(\widetilde{\lambda}\right)^{-1/\sigma-1} + \Lambda\eta\widetilde{\lambda}^{\eta-1}\right)} < 0.$$

Together with $\partial\,(\phi/\lambda)\,/\partial\widetilde{\lambda} \leq 0$ from above, this gives $\partial\,(\phi/\lambda)\,/\partial\Lambda \leq 0$. ∎

Second, the covariance between the symmetric function Λ of the spouses' wage rates and an individual's stochastic wage component z_i increases in the individual's relative wage-fixed effect in the household Ψ_i/Ψ_{-i}. This implies that the relative strength of the estimation bias also increases in the relative wage fixed effect.

Lemma 2 *The relative estimation bias in the household,* $\mathrm{bias}_i \,/\, \mathrm{bias}_{-i}$ *is an increasing function of the relative wage fixed effects,* Ψ_i/Ψ_{-i}. *Further,* $\mathrm{bias}_i \to 0$ *for* $\Psi_i \to 0$ *and for* $\Psi_{-i} \to \infty$.

Proof. From Proposition 1, it follows that the relative estimation bias in the household is given by $\mathrm{cov}\left(\phi/\lambda, \mathrm{E}\ln w_i' - \ln w_i\right) / \mathrm{cov}\left(\phi/\lambda, \mathrm{E}\ln w_{-i}' - \ln w_{-i}\right)$. Using the specification of the wage process, this is equivalent to

$$\frac{\mathrm{bias}_i}{\mathrm{bias}_{-i}} = \mathrm{cov}\left(\phi/\lambda, z_i'\right) / \mathrm{cov}\left(\phi/\lambda, z_{-i}'\right). \tag{51}$$

Combining $\partial\left(\phi/\lambda\right)/\partial\Lambda \leq 0$ and $\partial\Lambda/\partial z_i > 0$ gives $\partial\left(\phi/\lambda\right)/\partial z_i \leq 0$ and $\mathrm{cov}\left(\phi/\lambda, z_i\right) \leq 0$.

Now, consider the covariance between Λ and the individual stochastic wage component z_i. For household member i, $\mathrm{cov}\left(\Lambda, Z_i\right) = \mathrm{cov}\left(\Psi_i^{1+\eta} Z_i^{1+\eta} + Z_{-i}^{1+\eta}, Z_{-i}\right) = \Psi_i^{1+\eta} \cdot \mathrm{cov}\left(Z_i^{1+\eta}, Z_i\right) + \mathrm{cov}\left(Z_i^{1+\eta}, Z_{-i}\right) = \Psi_i^{1+\eta} \cdot \mathrm{cov}\left(Z_i^{1+\eta}, Z_i\right)$. Since $\mathrm{cov}\left(Z_i^{1+\eta}, Z_i\right)$ is an exogenous constant determined by the parameters of the wage process, $\mathrm{cov}\left(\Lambda, Z_i\right)$ is an increasing function of Ψ_i with $\mathrm{cov}\left(\Lambda, Z_i\right) \to 0$ for $\Psi_i \to 0$. For the other household member $-i$, we have $\mathrm{cov}\left(\Lambda, Z_{-i}\right) = \mathrm{cov}\left(Z_{-i}^{1+\eta}, Z_{-i}\right)$. Due to $\rho = 0$, and $\mathrm{var}\left(\varepsilon_1\right) = \mathrm{var}\left(\varepsilon_2\right)$, we can also use that $\mathrm{cov}\left(Z_{-i}^{1+\eta}, Z_{-i}\right) = \mathrm{cov}\left(Z_i^{1+\eta}, Z_i\right)$ which implies that

$$\frac{\mathrm{cov}\left(\Lambda, Z_i\right)}{\mathrm{cov}\left(\Lambda, Z_{-i}\right)} = \frac{\Psi_i^{1+\eta}}{\Psi_{-i}^{1+\eta}}. \tag{52}$$

Since the stochastic wage components are i.i.d., the correlation between current wages and the beginning-of-period asset level a, which is determined in the previous period, is zero. The proof continues by stating results conditional on the beginning-of-period asset holdings a and later continues by aggregating over a.

Generally, for a function $f(x)$, it holds that $\mathrm{cov}\left(f(x), x|a\right) = \mathrm{E}\left(\partial f/\partial x|a\right) \cdot \mathrm{cov}\left(x, y|a\right)$. Define $f(\Lambda) = \phi/\lambda$ and consider the covariance

$$\begin{aligned}\mathrm{cov}\left(f(\Lambda), Z_i|a\right) &= \mathrm{E}\left(\partial\left(\phi/\lambda\right)/\partial\Lambda|a\right) \cdot \mathrm{cov}\left(\Lambda, Z_i|a\right)\\ &= \mathrm{E}\left(\partial\left(\phi/\lambda\right)/\partial\Lambda|a\right) \cdot \mathrm{cov}\left(\Lambda, Z_i\right).\end{aligned}$$

The fact that $\partial\left(\phi/\lambda\right)/\partial\Lambda \leq 0$ implies that the first factor on the right-hand side is negative but it does not depend on the person index i.

Now, aggregating over the different a gives $\mathrm{cov}\left(f(\Lambda), Z_i\right) = \int \mathrm{cov}\left(f(\Lambda), Z_i|a\right)\ \mathrm{d}h(a) = \mathrm{cov}\left(\Lambda, Z_i\right) \cdot \int \mathrm{E}\left(\partial\left(\phi/\lambda\right)/\partial\Lambda|a\right)\ \mathrm{d}h(a)$ and hence

$$\frac{\mathrm{cov}\left(f(\Lambda), Z_i\right)}{\mathrm{cov}\left(f(\Lambda), Z_{-i}\right)} = \frac{\mathrm{cov}\left(\Lambda, Z_i\right) \cdot \int \mathrm{E}\left(\partial\left(\phi/\lambda\right)/\partial\Lambda|a\right)\ \mathrm{d}h(a)}{\mathrm{cov}\left(\Lambda, Z_i\right) \cdot \int \mathrm{E}\left(\partial\left(\phi/\lambda\right)/\partial\Lambda|a\right)\ \mathrm{d}h(a)} = \frac{\mathrm{cov}\left(\Lambda, Z_i\right)}{\mathrm{cov}\left(\Lambda, Z_i\right)} = \frac{\Psi_i^{1+\eta}}{\Psi_{-i}^{1+\eta}}, \tag{53}$$

where the final step uses (52).

Finally, we use $Z_i \approx 1 + z_i$ and, hence, $\text{cov}(\phi/\lambda, Z_g) \approx \text{cov}(\phi/\lambda, z_g)$ and combine (51) and (53) to

$$\frac{\text{bias}_i}{\text{bias}_{-i}} \approx \frac{\Psi_i^{1+\eta}}{\Psi_{-i}^{1+\eta}}.$$

From this, it follows that the estimation bias for household member approaches zero if either $\Psi_i \to 0$ or $\Psi_{-i} \to \infty$. Further, the estimation bias for household member i is strictly increasing in his or her relative wage fixed effect Ψ_i/Ψ_{-i}. ■

Lemma 3 *The relative contribution of household member i to household earnings is monotonically increasing in his or her relative wage fixed effect, Ψ_i/Ψ_{-i}.*

Proof. Consider the first-order conditions for labor supply, (8) and (9), to see $w_i n_i / (w_{-i} n_{-i}) = (w_i/w_{-i})^{1+\eta}$ for any combination of w_i, w_{-i}, a. Hence, for any asset level, relative earnings are an increasing function of relative wage rates. Taking logs gives $\ln w_i n_i - \ln w_{-i} n_{-i} = (1+\eta) \cdot (\ln w_1 - \ln w_2)$ and taking expectations gives

$$\text{E}(\ln w_1 n_1 - \ln w_2 n_2) = (1+\eta) \cdot (\text{E} \ln w_1 - \text{E} \ln w_2) = (1+\eta) \cdot \ln(\Psi_1/\Psi_2).$$

Hence, the mean earnings gap is an increasing function of the relative wage fixed effects Ψ_i/Ψ_{-i}. Finally, it is straightforward to show that the relation between the earnings gap and the earnings contribution is positive and monotonic. Define $v = \ln w_1 n_1 - \ln w_2 n_2$ and $x_1 = w_1 n_1 / (w_1 n_1 + w_2 n_2)$. Then $s = v/(v+1)$. ■

Proposition 4 *The estimation bias $(\widehat{\eta} - \eta)/\eta$ for an individual is monotonically related to the individual's percentage contribution to household earnings: The higher is the contribution to household earnings, the stronger is the estimation bias. The estimation bias converges to zero for individuals whose percentage contribution to household earnings converges to zero.*

Proof. Follows directly from Lemma 2 together with Lemma 3. ■